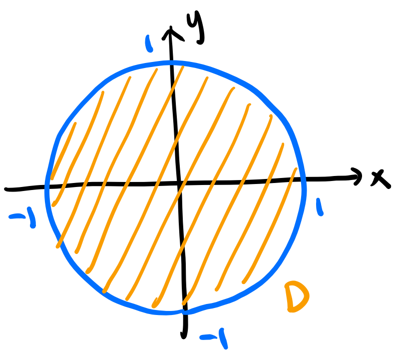
1. Consider the double integral *This problem is not well-formulated. See the note.

$$\int\int_D \frac{dA}{\sqrt{x^2+y^2}}.$$

(a) (7 points) Evaluate the double integral if D is the unit disc $x^2+y^2 \leq 1$.



In polar coordinates,

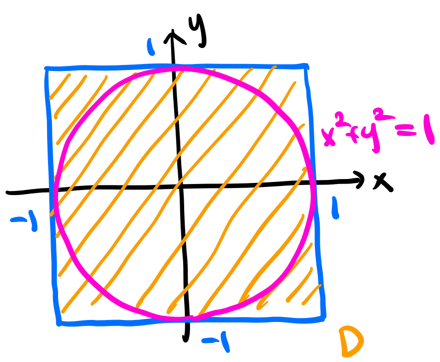
$D: 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq 1$

Jacobian

$$\iint_D \frac{1}{\sqrt{x^2+y^2}}\, dA = \int_0^{2\pi}\int_0^1 \frac{1}{r}\cdot r\, dr\, d\theta = \int_0^{2\pi}\int_0^1 1\, dr\, d\theta$$

$$= \int_0^{2\pi} 1\, d\theta = \boxed{2\pi}$$

(b) (3 points) Suppose now that D is the solid square with vertices at $(\pm 1, \pm 1)$. Is this double integral greater, equal to, or less than the answer to part (a)?



D_1: the unit disk $x^2+y^2 \leq 1$

$\Rightarrow$ D contains D_1

$\frac{1}{\sqrt{x^2+y^2}} > 0$ on D.

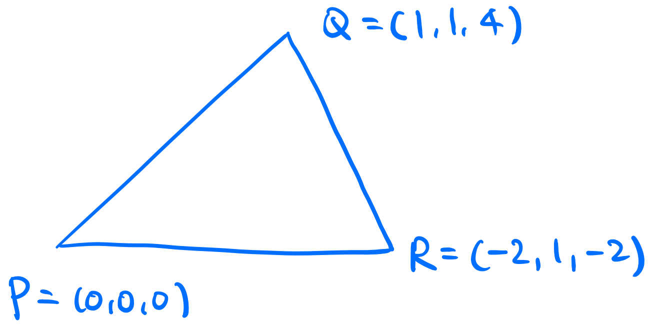
$$\Rightarrow \boxed{\iint_D \frac{1}{\sqrt{x^2+y^2}}\, dA > \iint_{D_1} \frac{1}{\sqrt{x^2+y^2}}\, dA \underset{\uparrow\ (a)}{=} 2\pi}$$

Note This problem is technically not well-formulated.

In fact, the integral $\iint_D \frac{1}{\sqrt{x^2+y^2}}\, dA$ does not make sense because $\frac{1}{\sqrt{x^2+y^2}}$ is not defined at $(0,0)$.

2. This problem has two parts.

(a) (6 points) Find the area of the triangle with vertices at $(0,0,0)$, $(1,1,4)$, and $(-2,1,-2)$.



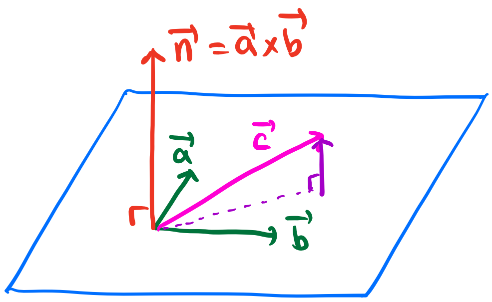
$$\overrightarrow{PQ} = (1,1,4)$$

$$\overrightarrow{PR} = (-2,1,-2)$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (-6,-6,3)$$

$$\Rightarrow \text{Area} = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2}\sqrt{(-6)^2+(-6)^2+3^2} = \boxed{\frac{9}{2}}$$

(b) (4 points) Suppose $\mathbf{a} = \mathbf{i}+\mathbf{j}+4\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i}+\mathbf{j}-2\mathbf{k}$. If $\mathbf{c} = 2\mathbf{i}+\mathbf{j}+3\mathbf{k}$, find the vector component of $\mathbf{c}$ that is perpendicular to the plane defined by $\mathbf{a}$ and $\mathbf{b}$. All vectors are assumed to originate at the origin.



$$\vec{a} = (1,1,4)$$

$$\vec{b} = (-2,1,-2)$$

$$\vec{c} = (2,1,3)$$

A normal vector of the plane is

$$\vec{n} = \vec{a} \times \vec{b} = (1,1,4) \times (-2,1,-2) = (-6,-6,3).$$

The vector component of $\vec{c}$ perpendicular to the plane is

$$\text{Proj}_{\vec{n}}\,\vec{c} = \left(\frac{\vec{n}\cdot\vec{c}}{|\vec{n}|^2}\right)\vec{n}.$$

$$\vec{n}\cdot\vec{c} = -6\cdot 2 - 6\cdot 1 + 3\cdot 3 = -9.$$

$$|\vec{n}|^2 = (-6)^2 + (-6)^2 + 3^2 = 81$$

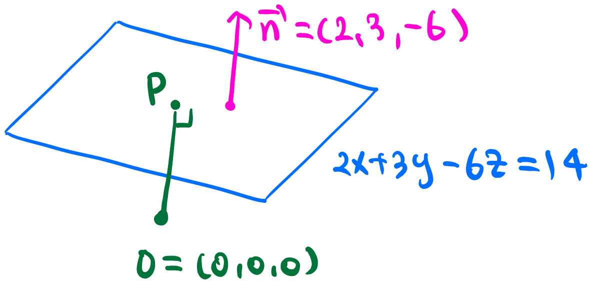
$$\Rightarrow \text{Proj}_{\vec{n}}\,\vec{c} = -\frac{9}{81}(-6,-6,3) = \boxed{\frac{1}{3}(2,2,-1)}$$

3. This problem has three parts.

(a) (2 points) Find the distance of the origin from the plane $2x + 3y - 6z = 14$.

$$\text{Distance} = \frac{|2\cdot 0 + 3\cdot 0 - 6\cdot 0 - 14|}{\sqrt{2^2 + 3^2 + (-6)^2}} = \frac{14}{7} = \boxed{2}$$

(b) (3 points) Find the point on the plane $2x + 3y - 6z = 14$ that is closest to the origin.



A normal vector of the plane is

$\vec{n} = (2, 3, -6)$.

$\overrightarrow{OP}$ is parallel to $\vec{n}$ with

$|\overrightarrow{OP}| \underset{\uparrow \text{(a)}}{=} 2$

The normal vector of $\vec{n}$ is $\dfrac{\vec{n}}{|\vec{n}|} = \dfrac{(2,3,-6)}{\sqrt{2^2+3^2+(-6)^2}} = \dfrac{1}{7}(2,3,-6)$

$\Rightarrow \overrightarrow{OP} = \pm \frac{2}{7}(2,3,-6). \rightsquigarrow P = \pm\frac{2}{7}(2,3,-6)$

At $\frac{2}{7}(2,3,-6)$: $2x+3y-6z = 2\cdot\frac{4}{7} + 3\cdot\frac{6}{7} + 6\cdot\frac{12}{7} = 14$. ✓

At $-\frac{2}{7}(2,3,-6)$: $2x+3y-6z = -2\cdot\frac{4}{7} - 3\cdot\frac{6}{7} - 6\cdot\frac{12}{7} \neq 14$.

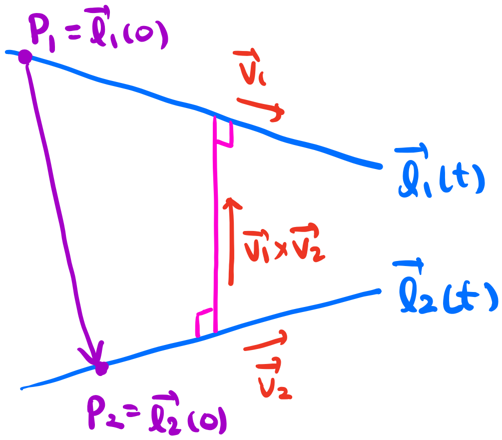
$\Rightarrow P = \boxed{\frac{2}{7}(2,3,-6)}$

<u>Note</u> You can also find the minimum of the distance

$f(x,y,z) = \sqrt{x^2+y^2+z^2}$ subject to $2x+3y-6z=14$

using the Lagrange multipliers.

(c) (5 points) The lines $(x, y, z) = (2, 2t + 1, t + 1)$ and $(x, y, z) = (3t - 2, 3, t + 1)$ do not intersect. Find the distance between the two lines.



$\vec{l}_1(t) = (2, 2t+1, t+1)$

$\rightsquigarrow$ a direction vector $\vec{v}_1 = (0, 2, 1)$

$\vec{l}_2(t) = (3t-2, 3, t+1)$

$\rightsquigarrow$ a direction vector $\vec{v}_2 = (3, 0, 1)$

The distance between the two lines is given by the length of the line segment which is perpendicular to both lines (represented by the pink line segment).

The direction of this line segment is given by

$\vec{v}_1 \times \vec{v}_2 = (0, 2, 1) \times (3, 0, 1) = (2, 3, -6)$

Take $P_1 = \vec{l}_1(0) = (2, 1, 1)$ and $P_2 = \vec{l}_2(0) = (-2, 3, 1)$

$\rightsquigarrow \overrightarrow{P_1P_2} = (-4, 2, 0)$.

The pink line segment corresponds to

$$\text{Proj}_{\vec{v}_1 \times \vec{v}_2} \overrightarrow{P_1P_2} = \left(\frac{(\vec{v}_1 \times \vec{v}_2) \cdot \overrightarrow{P_1P_2}}{|\vec{v}_1 \times \vec{v}_2|^2} \right) \vec{v}_1 \times \vec{v}_2$$

$(\vec{v}_1 \times \vec{v}_2) \cdot \overrightarrow{P_1P_2} = 2 \cdot (-4) + 3 \cdot 2 - 6 \cdot 0 = -2$.

$|\vec{v}_1 \times \vec{v}_2|^2 = 2^2 + 3^2 + (-6)^2 = 49$.

$\Rightarrow \text{Proj}_{\vec{v}_1 \times \vec{v}_2} \overrightarrow{P_1P_2} = -\frac{2}{49}(2, 3, -6)$

The distance is $\left| \text{Proj}_{\vec{v}_1 \times \vec{v}_2} \overrightarrow{P_1P_2} \right| = \frac{2}{49}\sqrt{2^2 + 3^2 + (-6)^2} = \boxed{\frac{2}{7}}$

4. This problem has three parts

(a) (2 points) Find $\frac{\partial u}{\partial r}$ if $u = x^2 + y$ and $x = r\cos\theta$, $y = r\sin\theta$.

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r} = \boxed{2x\cos\theta + 1\cdot\sin\theta}$$

chain rule

(b) (4 points) Find $\frac{\partial u}{\partial r}$ if $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$.

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r} = \boxed{f_x\cos\theta + f_y\sin\theta}$$

chain rule

(c) (4 points) Find $\frac{\partial u}{\partial x}$ if $u = g(r, \theta)$ and $x = r\cos\theta$, $y = r\sin\theta$.

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r} = \frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\sin\theta \quad \text{— } (*)$$

chain rule

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial \theta} = -\frac{\partial u}{\partial x} r\sin\theta + \frac{\partial u}{\partial y} r\cos\theta. \quad \text{— } (**)$$

$$(*) \times r\cos\theta : \quad r\cos\theta \frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} r\cos^2\theta + \frac{\partial u}{\partial y} r\cos\theta\sin\theta$$

$$(**) \times \sin\theta : \quad \sin\theta \frac{\partial u}{\partial \theta} = -\frac{\partial u}{\partial x} r\sin^2\theta + \frac{\partial u}{\partial r} r\cos\theta\sin\theta$$

$$\Rightarrow r\cos\theta \frac{\partial u}{\partial r} - \sin\theta \frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x} \cdot r$$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{1}{r}\left(r\cos\theta \frac{\partial u}{\partial r} - \sin\theta \frac{\partial u}{\partial \theta}\right)$$

$$= \cos\theta \frac{\partial u}{\partial r} - \frac{\sin\theta}{r} \cdot \frac{\partial u}{\partial \theta}$$

$$= \boxed{\cos\theta \cdot g_r - \frac{\sin\theta}{r} g_\theta}$$

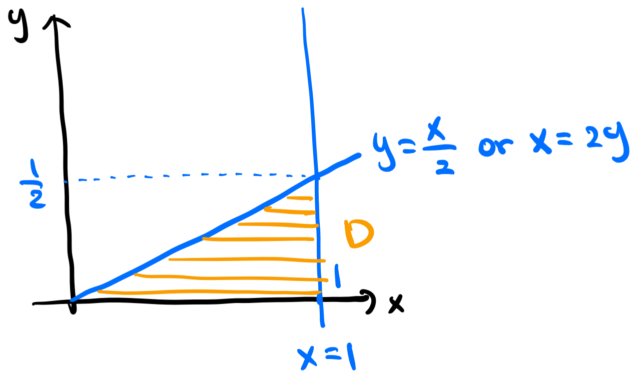
5. Both parts ask you to reverse the order of integration.

(a) (4 points) Rewrite the integral

$$\int_{x=0}^{1} \int_{y=0}^{x/2} f(x, y)\, dy\, dx$$

with x inner and y outer.

The domain is $D = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le 1,\ 0 \le y \le \frac{x}{2}\}$.



$D = \{(x,y) \in \mathbb{R}^2 : 0 \le y \le \frac{1}{2},\ 2y \le x \le 1\}$.

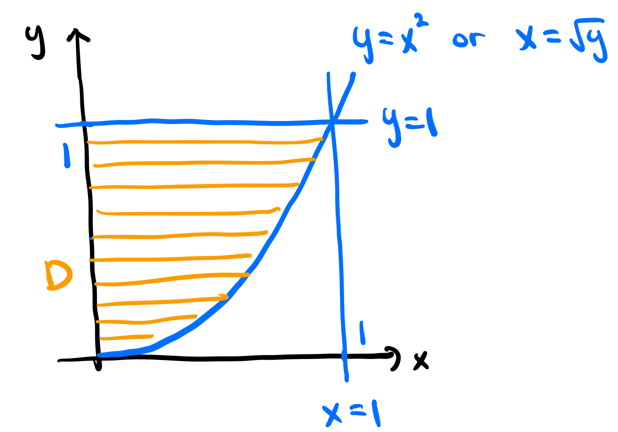
$$\Rightarrow \iint_D f(x,y)\, dA = \boxed{\int_0^{1/2} \int_{2y}^{1} f(x,y)\, dx\, dy}$$

(b) (6 points) Rewrite the integral

$$\int_{x=0}^{1} \int_{y=x^2}^{1} f(x, y)\, dy\, dx$$

with x inner and y outer.

The domain is $D = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le 1,\ x^2 \le y \le 1\}$.



$D = \{(x,y) \in \mathbb{R}^2 : 0 \le y \le 1,\ 0 \le x \le \sqrt{y}\}$

$$\Rightarrow \iint_D f(x,y)\, dA = \boxed{\int_0^1 \int_0^{\sqrt{y}} f(x,y)\, dx\, dy}$$

6. Consider the helix $(x, y, z) = (\cos t, \sin t, t)$, with t being the parameter.

(a) (2 points) If t is time and $(\cos t, \sin t, t)$ is the position of a particle at time t, find the magnitude of its acceleration.

$\vec{r}(t) = (\cos t, \sin t, t)$: position at time t.

$\vec{r}'(t) = (-\sin t, \cos t, 1)$: velocity at time t.

$\vec{r}''(t) = (-\cos t, -\sin t, 0)$: acceleration at time t.

$$|\vec{r}''(t)| = \sqrt{\cos^2 t + \sin^2 t + 0} = \boxed{1}$$

(b) (3 points) Find the length of the helix from $t = 0$ to $t = 2\pi$.

C: the curve parametrized by $\vec{r}(t)$ on $0 \le t \le 2\pi$.

$$\text{Length} = \int_C 1\, ds = \int_0^{2\pi} |\vec{r}'(t)|\, dt$$

$$|\vec{r}'(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$

$$\Rightarrow \text{Length} = \int_0^{2\pi} \sqrt{2}\, dt = \boxed{2\sqrt{2}\,\pi}$$

(c) (5 points) Assume that the density (mass per unit length) of the helix from $t = 0$ to $t = 2\pi$ is constant and equal to 1. Find $\bar{z}$, the z-coordinate of the center of mass of the part of the helix from $t = 0$ to $t = 2\pi$.

Density $\rho(x, y, z) = 1$.

$$\text{Mass } m = \int_C \rho(x,y,z)\, ds = \int_C 1\, ds \underset{\text{(b)}}{=} 2\sqrt{2}\,\pi$$

$$\bar{z} = \frac{1}{m} \int_C z\,\rho(x,y,z)\, ds = \frac{1}{2\sqrt{2}\,\pi} \int_C z\, ds$$

$$= \frac{1}{2\sqrt{2}\,\pi} \int_0^{2\pi} z(\vec{r}(t))\, |\vec{r}'(t)| = \frac{1}{2\sqrt{2}\,\pi} \int_0^{2\pi} t \cdot \sqrt{2}\, dt$$

$$= \frac{1}{2\sqrt{2}\,\pi} \cdot \frac{\sqrt{2}}{2} t^2 \Big|_{t=0}^{t=2\pi} = \boxed{\pi}$$

7. In each part, the curve C is assumed to be counterclockwise. Evaluate

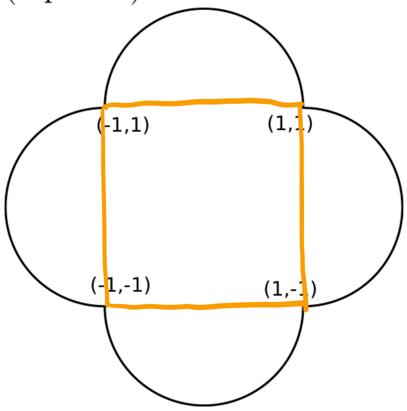
$$\int_C y\,dx$$

for the following C:

(a) (2 points) C is the circle $x^2 + y^2 = 4$.

(b) (4 points) C is the square with vertices at $(\pm 1, \pm 1)$.

(c) (4 points) C is the curve below (the arcs are semicircles):



$$\int_C y\,dx = \int_C y\,dx + 0\,dy \Rightarrow P = y,\ Q = 0.$$

For each C, let D be the region enclosed by C.

$\Rightarrow \partial D = C$ is positively oriented.

$$\int_C y\,dx = \int_{\partial D} y\,dx \underset{\text{Green's thm}}{=} \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dA = \iint_D -1\,dA = -\text{Area}(D)$$

(a) $D : x^2+y^2 \le 4 \rightsquigarrow$ a disk of radius 2.

$$-\text{Area}(D) = -\pi \cdot 2^2 = \boxed{-4\pi}$$

(b) $D = [-1,1] \times [-1,1] \Rightarrow -\text{Area}(D) = -2\cdot 2 = \boxed{-4}$

(c) $-\text{Area}(D) = -(\underbrace{4 \cdot \tfrac{1}{2}\pi \cdot 1^2}_{\text{semidisks}} + \underbrace{2 \cdot 2}_{\text{square}}) = \boxed{-4-2\pi}$

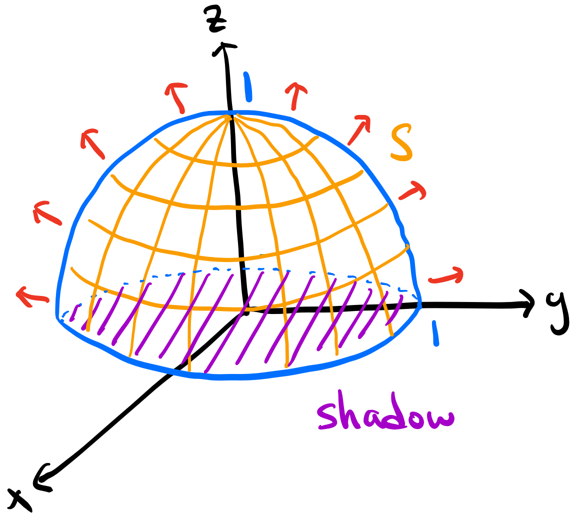
8. Let S be the hemispherical surface $x^2 + y^2 + z^2 = 1$ between the planes $z = 0$ and $z = 1$. The normal to the surface or $d\mathbf{S}$ is assumed to be pointing out of the center of the hemisphere.

(a) (5 points) Find the flux

$$\int\int_S \mathbf{F} \cdot d\mathbf{S}$$

with $\mathbf{F} = z\mathbf{k}$.

Sol 1 (Using a parametrization)



$x^2+y^2+z^2=1,\ z \geq 0 \rightsquigarrow z = \sqrt{1-x^2-y^2}$

Shadow on the xy-plane:

$D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1\}$.

$\Rightarrow 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq 1$ in polar coordinates

$\rightsquigarrow$ S is parametrized by

$\vec{r}(x,y) = (x, y, \sqrt{1-x^2-y^2})$ on D.

$$\vec{r}_x = \left(1, 0, -\frac{x}{\sqrt{1-x^2-y^2}}\right),\quad \vec{r}_y = \left(0, 1, -\frac{y}{\sqrt{1-x^2-y^2}}\right)$$

$$\vec{r}_x \times \vec{r}_y = \left(\frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, \underset{\overset{\vee}{0}}{1}\right) : \text{oriented upward}.$$

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_D \vec{F}(\vec{r}(x,y)) \cdot (\vec{r}_x \times \vec{r}_y)\, dA.$$

$$\vec{F}(x,y,z) = (0,0,z) \rightsquigarrow \vec{F}(\vec{r}(x,y)) = (0,0,\sqrt{1-x^2-y^2})$$

$$\vec{F}(\vec{r}(x,y)) \cdot (\vec{r}_x \times \vec{r}_y) = \sqrt{1-x^2-y^2}$$

polar coords

Jacobian

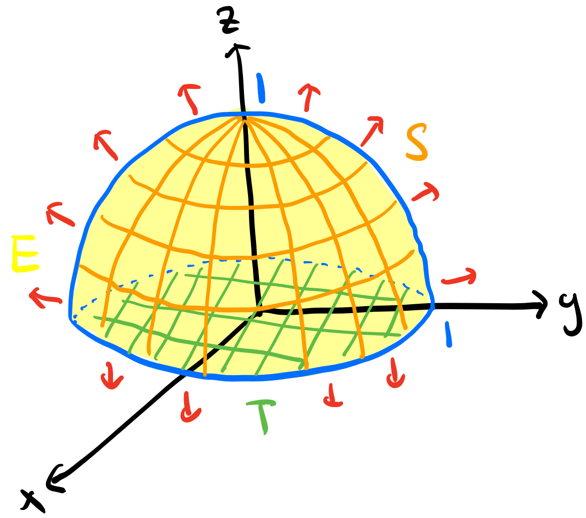
$$\Rightarrow \iint_S \vec{F}\cdot d\vec{S} = \iint_D \sqrt{1-x^2-y^2}\, dA = \int_0^{2\pi}\int_0^1 \sqrt{1-r^2}\cdot r\, dr\, d\theta$$

$(u = 1-r^2 \Rightarrow du = -2r\,dr)$

$$= \int_0^{2\pi}\int_1^0 -\frac{\sqrt{u}}{2}\, du\, d\theta = \int_0^{2\pi} -\frac{u^{3/2}}{3}\Big|_{u=1}^{u=0} d\theta$$

$$= \int_0^{2\pi} \frac{1}{3}\, d\theta = \boxed{\frac{2}{3}\pi}$$

Sol 2 (Using the divergence theorem)



T: $x^2+y^2 \le 1$ and $z=0$ with downward orientation

E: the solid bounded by S and T

$\Rightarrow \partial E = S \cup T$ is oriented outward.

$\vec{F} = (0,0,z) \Rightarrow \mathrm{div}(\vec{F}) = 0+0+1 = 1$.

$$\iint_{\partial E} \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot d\vec{S} + \iint_T \vec{F}\cdot d\vec{S}.$$

$$\iint_{\partial E} \vec{F}\cdot d\vec{S} \underset{\text{divergence thm.}}{=} \iiint_E \mathrm{div}(\vec{F})\,dV = \iiint_E 1\,dV$$

$$= \mathrm{Vol}(E) = \frac{1}{2}\cdot\frac{4}{3}\pi\cdot 1^3 = \frac{2}{3}\pi$$

$$\Rightarrow \iint_S \vec{F}\cdot d\vec{S} = \frac{2}{3}\pi - \iint_T \vec{F}\cdot d\vec{S}.$$

T lies on the xy-plane with downward orientation.

$\leadsto$ The unit normal vector on T is $\vec{n} = (0,0,-1)$

$$\vec{F}\cdot\vec{n} = (0,0,z)\cdot(0,0,-1) = -z \underset{z=0 \text{ on } T}{=} 0$$

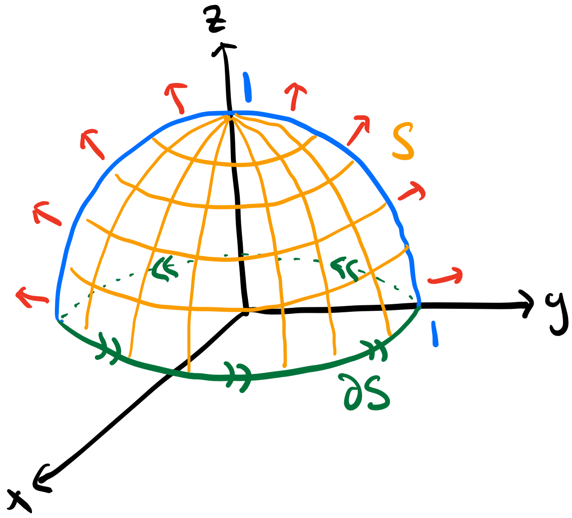
$$\Rightarrow \iint_T \vec{F}\cdot d\vec{S} = \iint_T \vec{F}\cdot\vec{n}\,dS = 0$$

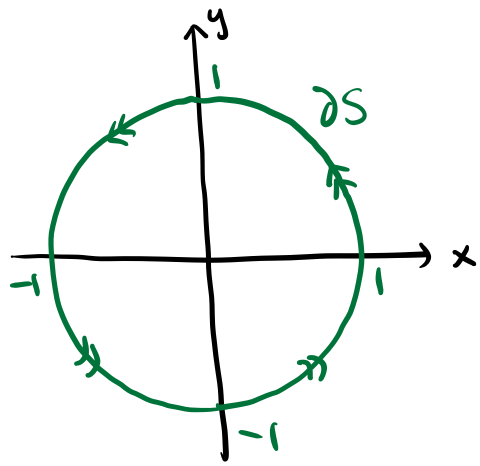
$$\Rightarrow \iint_S \vec{F}\cdot d\vec{S} = \frac{2}{3}\pi - 0 = \boxed{\frac{2}{3}\pi}$$

(b) (5 points) Find the flux

$$\int\int_S \text{curl}\,\mathbf{F}\cdot d\mathbf{S}$$

with $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$.





The positive orientation for ∂S is counterclockwise.

$\rightsquigarrow$ ∂S is parametrized by

$\vec{r}(t) = (\cos t, \sin t, 0)$ on $0 \le t \le 2\pi$.

$$\iint_S \text{curl}(\vec{F})\cdot d\vec{S} \underset{\text{Stokes thm.}}{=} \int_{\partial S} \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt.$$

$\vec{F}(x,y,z) = (-y, x, z) \rightsquigarrow \vec{F}(\vec{r}(t)) = (-\sin t, \cos t, 0)$

$\vec{r}\,'(t) = (-\sin t, \cos t, 0)$

$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = \sin^2 t + \cos^2 t + 0 = 1.$

$$\Rightarrow \iint_S \text{curl}(\vec{F})\cdot d\vec{S} = \int_0^{2\pi} 1\,dt = \boxed{2\pi}$$

<u>Note</u> You can also use the divergence theorem by closing the base T as in (a).

9. The position vector is given by $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(a) (3 points) Let $\mathbf{F} = |\mathbf{r}|^2\mathbf{r}$. Find div $\mathbf{F}$.

$$\vec{r} = (x, y, z) \rightsquigarrow |\vec{r}| = \sqrt{x^2+y^2+z^2}$$

$$\vec{F} = \left((x^2+y^2+z^2)x,\ (x^2+y^2+z^2)y,\ (x^2+y^2+z^2)z\right)$$

$$= (x^3+xy^2+xz^2,\ x^2y+y^3+yz^2,\ x^2z+y^2z+z^3)$$

$$\operatorname{div}(\vec{F}) = 3x^2+y^2+z^2+x^2+3y^2+z^2+x^2+y^2+3z^2 = \boxed{5(x^2+y^2+z^2)}$$

(b) (3 points) Again let $\mathbf{F} = |\mathbf{r}|^2\mathbf{r}$. Find the outward flux

$$\int\int_S \mathbf{F} \cdot d\mathbf{S}$$

with S being the surface of the cube with vertices at $(\pm1, \pm1, \pm1)$.

$$E = [-1,1] \times [-1,1] \times [-1,1]$$

$\Rightarrow \partial E = S$ is oriented outward.

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_{\partial E} \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{div}(\vec{F})\,dV$$

$$\underset{(a)}{\uparrow}= \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 5x^2+5y^2+5z^2\,dz\,dy\,dx$$

$$= \int_{-1}^{1}\int_{-1}^{1} 5x^2z+5y^2z+\frac{5}{3}z^3 \Big|_{z=-1}^{z=1} dy\,dx$$

$$= \int_{-1}^{1}\int_{-1}^{1} 10x^2+10y^2+\frac{10}{3}\,dy\,dx$$

$$= \int_{-1}^{1} 10x^2y+\frac{10}{3}y^3+\frac{10}{3}y \Big|_{y=-1}^{y=1} dx$$

$$= \int_{-1}^{1} 20x^2+\frac{40}{3}\,dx = \frac{20}{3}x^3+\frac{40}{3}x \Big|_{x=-1}^{x=1} = \boxed{40}$$

(c) (4 points) Now suppose $\mathbf{F} = \frac{\mathbf{r}}{|\mathbf{r}|^3}$. Find the outward flux

$$\int\int_S \mathbf{F} \cdot d\mathbf{S}$$

with S being the surface of the cube with vertices at $(\pm 1, \pm 1, \pm 1)$.

$$\vec{F} = \left(\frac{x}{(x^2+y^2+z^2)^{3/2}}, \frac{y}{(x^2+y^2+z^2)^{3/2}}, \frac{z}{(x^2+y^2+z^2)^{3/2}} \right)$$

$\rightsquigarrow$ the inverse square field.

S is a closed surface which encloses the origin with outward orientation.

$$\Rightarrow \iint_S \vec{F} \cdot d\vec{S} = \boxed{4\pi}$$

(see Fact 4(4) in the Final exam facts note)